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V_∞ Leveraging for Interplanetary Missions: Multiple-Revolution Orbit Techniques

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The concept of a delta-velocity Earth gravity-assist trajectory is generalized to include multiple revolutions of both the spacecraft and the Earth about the sun. The concept is also generalized to include trajectories to destinations inside the orbit of the Earth. Both of these generalizations fall under the comprehensive concept of V_∞ leveraging. Analytic results predict the existence of low-launch-energy trajectories to asteroids and Mercury. Numerical results confirm these predictions. These low-launch-energy trajectories represent excellent opportunities for low-cost missions using small launch vehicles.

Introduction

THE term V_∞ leveraging refers to the use of a relatively small deep-space maneuver to modify the V_∞ (excess hyperbolic velocity) at a body. This maneuver, in conjunction with a gravity assist at the body, reduces the launch energy requirements and the total ΔV for a mission. A typical example of V_∞ leveraging is the delta-velocity Earth gravity-assist (ΔV-EGA) trajectory introduced by Hollenbeck.[1] However, because the technique may be used with any gravity-assist body, the term V_∞ leveraging was introduced to apply to the general case and to be consistent with other mechanical terms that are in use. (The first documented use of the term V_∞ leveraging is by Williams[2] when describing a triple Venus flyby trajectory to Saturn. The first general use of the term as described here is by Sims and Longuski.[3]) For example, the term orbit pumping is used by Beckman and Smith[4] in reference to increasing (pumping up) or decreasing (pumping down) the energy (and size) of an orbit relative to the primary body by a gravity-assist flyby of a secondary body. The term orbit cranking is introduced by Roberts and Uphoff[5] to describe the process of changing the orbital inclination relative to the primary body by a gravity assist with a secondary body.

We develop the analytic theory of V_∞ leveraging with Earth. We delineate the numerical procedure we use to compute the performance of these trajectories, and then we describe their characteristics, including why the term V_∞ leveraging is appropriate. Even though the exact two-body equations are relatively simple, solving them requires an iterative algorithm because of their transcendental nature. (In a previous paper, Sims and Longuski[3] develop a closed-form, approximate solution for the performance of V_∞ leveraging. A useful analytic approach involving Jacobi's integral to estimate the magnitude of the deep-space ΔV, first given by Sweetser,[6] is incorporated in the analysis and compared with another method.)

We use numerical techniques to extend the ΔV-EGA concept to include multiple revolutions of the spacecraft and the Earth. We also introduce analogous trajectories with heliocentric orbits inside the Earth's orbit. The techniques are not unique to Earth; they can be applied using any planet as the secondary body or even to systems with a primary body other than the sun. The usefulness and accuracy of the analytic theory is demonstrated by finding trajectories to an asteroid and to Mercury. Finally, we explore the possibility of replacing the deep-space ΔV with a gravity-assist maneuver.

Exterior ΔV-EGA

In an exterior ΔV-EGA trajectory as examined by Hollenbeck,[1] a spacecraft is launched from Earth into a heliocentric orbit with a period slightly greater than an integer number of years and a perihelion radius equal to 1 AU (assuming a circular Earth orbit). At aphelion, a (tangential) retrograde ΔV is applied to lower the perihelion to intercept the Earth nontangentially with a V_∞ higher than that of launch. Kepler's equation is used to compute the time to intercept, which leads to an iterative procedure to determine the aphelion ΔV. This maneuver enables the Earth to be used as a gravity-assist body to increase the heliocentric energy of the spacecraft. As shown in Fig. 1a, the reencounter with Earth can occur either before or after perihelion of the new orbit. We will refer to the orbit that has a period equal to an integer number of years as the nominal orbit. For the nominal orbit, no maneuver is required to intercept the Earth, and the V_∞ is the same on return as it was at launch.

Hollenbeck[1] presented only the cases in which the period of the nominal orbit is an integer number of years. However, the nominal orbit period of the spacecraft does not have to be an integer multiple of the orbit period of the Earth (or other secondary body), as long as interception occurs in an integer number of periods. For example, the nominal orbit for a ΔV-EGA trajectory can have a period of 1.5 years, in which case the interception will take place after 3 years. Whereas the previous ΔV-EGA trajectories were adequately designated by a single number, we will introduce a new way to designate the type of ΔV-EGA, inasmuch as we are now considering cases in which both the Earth and the spacecraft make multiple revolutions. The new designation is as follows:

$$K : L(M)^{\pm}$$

where

K	=	number of Earth orbit revolutions
L	=	number of spacecraft orbit revolutions
M	=	spacecraft orbit revolution on which the maneuver is performed
$+, -$	=	Earth encounter just after (before) the spacecraft passes the line of apsides

The number in parentheses was added because the maneuver can be performed on any of the spacecraft revolutions. This number is optional and may be omitted when not important to the discussion or if the number of spacecraft revolutions is one. For example, what was previously denoted a $3^{+}\Delta V$-EGA will now be a $3{:}1(1)^{+}$ or just $3{:}1^{+}\Delta V$-EGA, and the ΔV-EGA trajectory described earlier is a 3:2(1) or 3:2(2) ΔV-EGA, depending on the revolution on which the maneuver is performed. We note that the period of the nominal orbit of the spacecraft (in years) can be obtained by dividing the first number, Earth revolutions, by the second number, spacecraft

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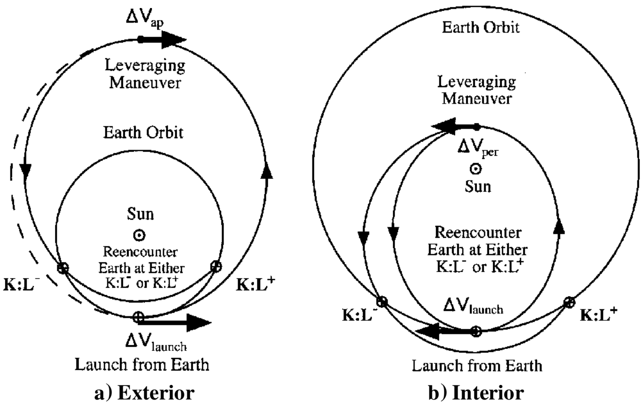

Fig. 1 Δ V-EGA trajectories.

revolutions (i.e., K/L). We also note that the first number (K) gives the approximate time in years between launch from Earth and the Earth gravity assist.

We use the following numerical algorithm to calculate the characteristics and performance of exterior ΔV-EGA trajectories. (More details of the derivation of the equations are provided by Sims.[7]) Starting with a value of the launch V_∞ slightly larger than that for the nominal orbit, we set the $\boldsymbol{V}_\infty$ parallel to the velocity of the Earth ($\boldsymbol{V}_E$). In this case, the spacecraft commences at perihelion ($r_p = 1$ AU) of a heliocentric orbit with velocity

$$V_p = V_E + V_\infty \tag{1}$$

Knowing r_p and V_p, we can determine the aphelion radius and velocity, r_a and V_a, from the following equations:

$$r_a = \frac{r_p^2 V_p^2}{2\mu_{\text{sun}} - r_p V_p^2} \tag{2}$$

where μ_{sun} is the gravitational parameter of the sun, and

$$V_a = V_p r_p / r_a \tag{3}$$

The period of the initial orbit, given by

$$P_l = \pi \sqrt{\frac{(r_a + r_p)^3}{2\mu_{\text{sun}}}} \tag{4}$$

is slightly longer than the period of the nominal orbit. Hence, if we propagate the orbit to the next perihelion, the Earth would not be located there. Thus, we guess a value for the maneuver at aphelion, ΔV_{ap}, necessary to reencounter the Earth. The maneuver is taken to be aligned with $\boldsymbol{V}_a$, and so the aphelion velocity of the return orbit, V_{ar}, is

$$V_{\text{ar}} = V_a - \Delta V_{\text{ap}} \tag{5}$$

The aphelion radius remains the same; however, the perihelion radius is smaller.

With the aphelion velocity and radius in hand, we can determine additional properties of the return orbit. The semimajor axis, period, and eccentricity of the return orbit are given by

$$a_r = \left[(2/r_a) - \left(V_{\text{ar}}^2/\mu_{\text{sun}}\right)\right]^{-1} \tag{6}$$

$$P_r = 2\pi\sqrt{a_r^3/\mu_{\text{sun}}} \tag{7}$$

$$e_r = (r_a/a_r) - 1 \tag{8}$$

We now check the location of the Earth when the spacecraft crosses the Earth's orbit, that is, when the radius of the return orbit equals the radius of Earth's orbit ($r_E = 1$ AU). This occurs at two points on the return orbit: once before perihelion and once after. The velocity at these two potential encounter points is given by

$$V_e = \sqrt{\mu_{\text{sun}}[(2/r_E) - (1/a_r)]} \tag{9}$$

The flight-path angle of the encounter γ_e, obtained from

$$\cos\gamma_e = \frac{r_a V_{\text{ar}}}{r_E V_e} \tag{10}$$

will be a positive value at the crossing point after perihelion and a negative value before perihelion. The true anomaly θ^* and eccentric anomaly E can be determined from the following equations:

$$\cos\theta_e^* = \frac{\left[a_r\left(1 - e_r^2\right)/r_E\right] - 1}{e_r} \tag{11}$$

and

$$\tan(E_e/2) = \sqrt{(1 - e_r)/(1 + e_r)}\tan\left(\theta_e^*/2\right) \tag{12}$$

We use Kepler's equation to determine the length of time from perihelion t_{ep}:

$$t_{ep} = \sqrt{a_r^3/\mu_{\text{sun}}}(E_e - e_r \sin E_e) \tag{13}$$

which is negative for a crossing point before perihelion. For the case of a single spacecraft revolution (i.e., $L = 1$), the flight time from launch to the crossing point is given by

$$T_e = (P_l/2) + (P_r/2) + t_{ep} \tag{14}$$

(For more than one spacecraft revolution, we add multiples of P_l and P_r as appropriate.) During that time the Earth has traveled a total angular distance of

$$\upsilon_E = T_e(2\pi/\text{year}) \tag{15}$$

resulting in a true anomaly of θ_E^*,

$$\theta_E^* = \upsilon_E - 2\pi K \tag{16}$$

relative to its location at launch. We now check to see whether the Earth is at the crossing point at the same time as the spacecraft by computing the difference in true anomaly $\Delta\theta^*$:

$$\Delta\theta^* = \theta_e^* - \theta_E^* \tag{17}$$

If $\Delta\theta^*$ is not zero, then we adjust the value of ΔV_{ap} and repeat the computations until $\Delta\theta^*$ is zero. (We use a numerical tolerance of 10^{-5}.) This procedure is equivalent to solving a transcendental equation in ΔV_{ap}. For a given value of the launch V_∞, the ΔV_{ap} required to reencounter the Earth before perihelion is different from after perihelion. Thus, this procedure is performed separately for the + and − type trajectories, and the characteristics of the two types of trajectories are slightly different. Once the correct ΔV_{ap} has been determined, we continue with the gravity assist at Earth.

Figure 2 shows the aphelion radius of the spacecraft orbit after the Earth flyby as a function of total ΔV. (Here we restrict our discussion to the exterior ΔV-EGAs, which have aphelion radii greater than 1 AU in Fig. 2.) For the thin solid lines, the total ΔV consists of the launch from an Earth parking orbit (circular, 185-km altitude) and the ΔV_{ap}. The minimum flyby altitude is 200 km; however, near the low end of the curves, maximum aphelion radius is attained with higher flyby altitudes. Figure 2 shows that + type trajectories have a larger maximum final aphelion radius and better performance near the maximum. We note, however, that − type trajectories require slightly less total ΔV for smaller final aphelion radii. This observation, which could be important in some applications, is not apparent in the work by Hollenbeck.[1]

The dashed lines in Fig. 2 represent the performance that can be achieved with an additional maneuver immediately after the Earth gravity assist. The dashed lines originate from the point on each ΔV-EGA curve beyond which it is more efficient to add ΔV after the Earth flyby instead of incorporating it into the ΔV-EGA (i.e., moving to the right on the ΔV-EGA curve).

The direct launch curve (thick solid line) is shown for comparison. As the figure indicates, ΔV-EGAs can significantly reduce the total ΔV required to reach a given aphelion radius. This reduction in

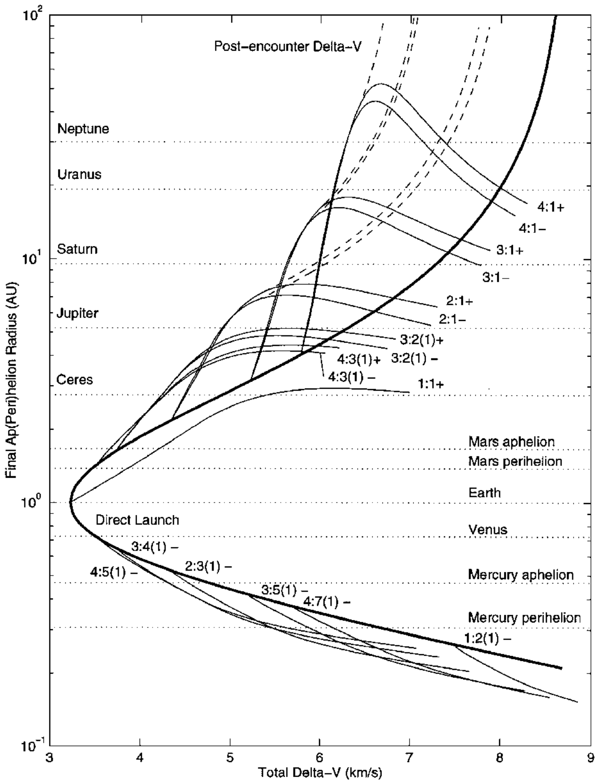

Fig. 2 ΔV-EGA performance.

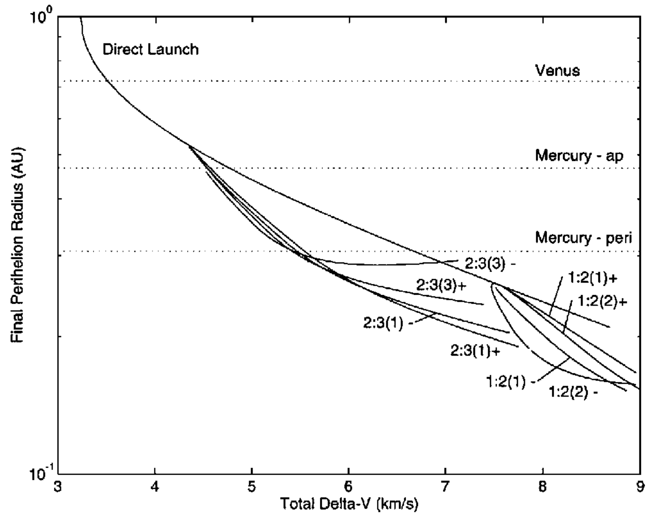

Fig. 3 Comparison of interior ΔV-EGAs: + vs − and revolution of maneuver.

total ΔV comes with the cost of a longer flight time and larger postlaunch ΔV. For example, a trajectory launched directly from Earth to Saturn requires a launch V_∞ of 10.3 km/s. The launch ΔV in this case is 7.28 km/s, which is the total ΔV because there are no postlaunch maneuvers. For a $3{:}1^-$ ΔV-EGA to Saturn, the launch V_∞ is 6.95 km/s, which is equivalent to a launch ΔV of 5.23 km/s. The postlaunch ΔV consists entirely of an aphelion ΔV of 0.39 km/s resulting in a total ΔV of 5.62 km/s. The time of flight from Earth launch to the flyby of Earth is 2.89 years. Hence, in this example the total ΔV has been reduced by 1.66 km/s with an increase in flight time of about 2.9 years.

As can be seen in Fig. 2, the 3:2 ΔV-EGAs have better performance than those with a single spacecraft orbit revolution for final aphelion radii between approximately 1.6 and 4.3 AU. This is a significant observation because the orbits of many of the major asteroids lie in this range. The 3:2(2) ΔV-EGAs (not shown) have a very slight performance advantage over 3:2(1) ΔV-EGAs for final aphelion radii below about 3 AU, corresponding to a total ΔV of around 4.3 km/s. The 4:3(1) ΔV-EGAs have even better performance for final aphelion radii below about 2.4 AU. The fact that the 3:2 and 4:3 trajectories remain below about 2 AU for an extended time could be an advantage for certain missions. We recall that the first number in the designation of the type of ΔV-EGA gives the approximate time in years between launch from Earth and the reencounter with Earth. Thus, for example, the 4:3 ΔV-EGAs take about 1 year longer than the 3:2 ΔV-EGAs (i.e., 4 vs 3 years).

Interior ΔV-EGA

In addition to using heliocentric orbits larger than the orbit of Earth, we can use smaller orbits. For example, a spacecraft is launched from Earth into a heliocentric orbit with a period of about $\frac{1}{2}$ year and an aphelion radius equal to 1 AU. (This is designated a 1:2 ΔV-EGA.) At perihelion a ΔV is applied to raise the aphelion to intercept the Earth nontangentially (see Fig. 1b). This enables the Earth to be used as a gravity-assist body. We call this type of trajectory an interior ΔV-EGA. The − or + now indicates Earth encounter before or after aphelion, respectively.

To calculate the characteristics and performance of interior ΔV-EGA trajectories, we use a procedure analogous to that described for exterior ΔV-EGAs (where, in general, the roles of aphelion and perihelion are reversed). Figure 2 shows the perihelion radius of the orbit after the Earth flyby as a function of total ΔV for several different types of interior ΔV-EGAs. (The interior ΔV-EGAs have perihelion radii less than 1 AU in Fig. 2.) Trajectories with periods less than 0.35 year cannot be achieved by launching from Earth because the minimum aphelion radius is 1 AU. Thus, for example, 1:3 and 1:4 ΔV-EGAs are not possible. The interior $1{:}1^+$ ΔV-EGA is not shown in Fig. 2, although it has a slight performance advantage over a direct launch for a small range of total ΔV. By comparison, the exterior $1{:}1^+$ ΔV-EGA has no performance advantage over a direct launch, as can be seen in Fig. 2. Numerical calculations demonstrate that $1{:}1^-$ ΔV-EGAs (interior or exterior) are not possible.

As with exterior ΔV-EGAs, the − type have slightly better performance on the first portion of each curve (the lower total ΔV end). On the same end of each curve, ΔV-EGAs with maneuvers on the last spacecraft revolution have the best performance. Both of these characteristics can be seen in Fig. 3. They occur with exterior ΔV-EGAs also; however, the effect is more apparent in the case of interior ΔV-EGAs, partly due to the log scale of the vertical axis.

Other Characteristics of ΔV-EGA Trajectories

The effectiveness of V_∞ leveraging is shown in Fig. 4 for exterior ΔV-EGAs. The increase in V_∞ at Earth is plotted vs the aphelion ΔV. As an example, for an aphelion ΔV of 1 km/s for a 3:1 ΔV-EGA, the V_∞ at Earth increases by about 8 km/s. If the ΔV of 1 km/s were applied at launch instead, the increase in V_∞ would be only 1.7 km/s. This example demonstrates the increased effectiveness of the maneuver when placed at aphelion and illustrates why the term V_∞ leveraging is appropriate. The V_∞ leveraging effectiveness for interior ΔV-EGAs is approximately the same as that for 3:2 and 4:3 ΔV−EGAs.[7,8]

The launch V_∞ and deep-space ΔV are shown in Figs. 5 and 6, respectively. To achieve a given aphelion or perihelion radius, a + type trajectory requires a slightly higher launch V_∞ and a slightly smaller deep-space ΔV than a − type trajectory. In many applications, minimizing the magnitude of the deep-space maneuver may be a more important consideration than minimizing the launch energy or the total ΔV, which includes the launch ΔV.

If the deep-space ΔV can be replaced with a gravity assist from another body, the total ΔV can be reduced even further over that of a direct launch. The total ΔV in this case consists entirely of the relatively small launch ΔV. We will see examples of this in the applications section where we use a Mars gravity assist to replace the deep-space maneuver.

We see from Fig. 4 that the V_∞ at Earth return increases monotonically with the aphelion ΔV, whereas Fig. 2 shows that the final

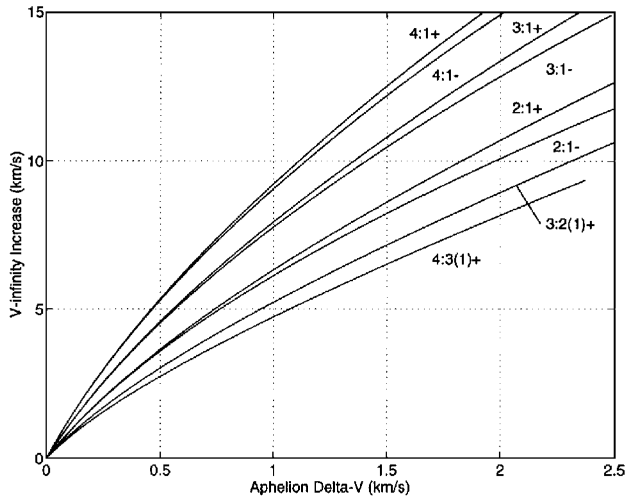

Fig. 4 Effectiveness of V_∞ leveraging for exterior ΔV-EGA trajectories.

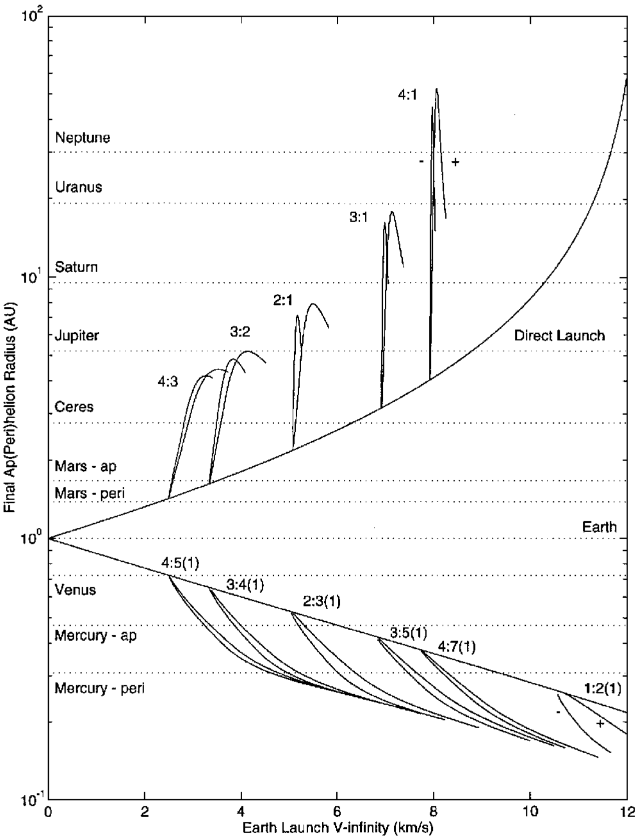

Fig. 5 Launch V_∞ requirements for ΔV-EGA trajectories.

aphelion radius reaches a maximum and then decreases with increasing ΔV (assuming no ΔV is applied after the flyby). The reason for this phenomenon is that the amount the $\boldsymbol{V}_\infty$ vector can be turned by the gravity of a flyby body decreases as the V_∞ increases. The tradeoff between V_∞ and turn angle is described in detail in Refs. 3 and 7. An analytic approximation of ΔV-EGA trajectories is also presented in these references, along with an analysis of exterior delta-velocity Venus gravity-assist (ΔV-VGA) trajectories and trajectories with multiple Venus flybys.

One method of circumventing the turn-angle limitation is to use aerogravity assist.[3,7] In this method, a lifting body flies through the atmosphere of the planet (Earth in this case) to turn the $\boldsymbol{V}_\infty$ in any desired direction. References 3 and 7 discuss the tremendous advantage possible using aerogravity assist, should this technology become available.

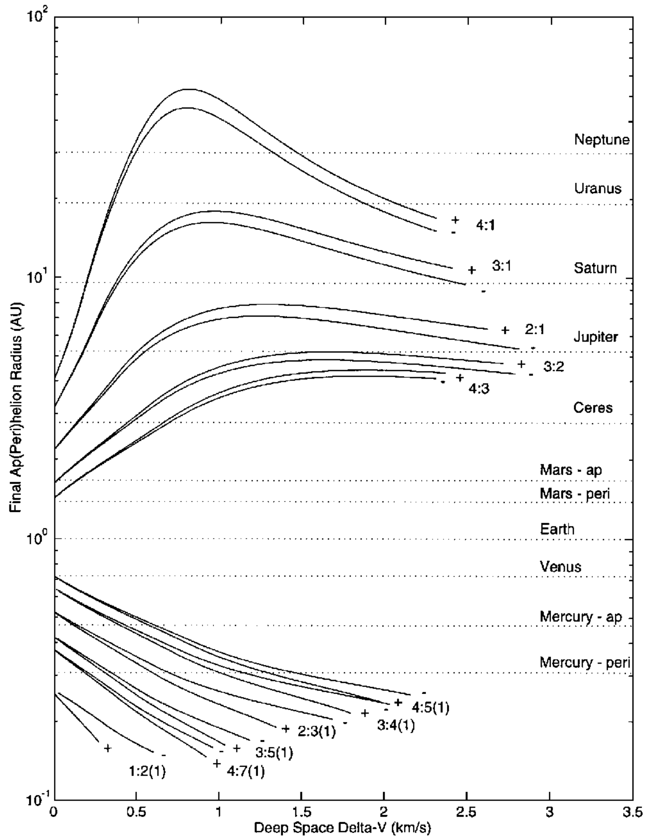

Fig. 6 Deep-space ΔV for ΔV-EGA trajectories.

Applications

A circular Earth orbit is assumed in assessing the potential aphelion and perihelion radii that can be reached by ΔV-EGA trajectories. To determine how well the analysis predicts the characteristics of interplanetary trajectories for a more realistic orbit of the Earth, we use an automated version of the Satellite Tour Design Program (STOUR)[2,9] to perform broad searches for patched-conic gravity-assist trajectories. The trajectories are optimized using a Jet Propulsion Laboratory program, Mission Design and Analysis Software (MIDAS).[10] MIDAS minimizes total ΔV while using patched-conic trajectory simulation. The program is capable of shifting trajectory event times such as launch date, arrival date, and flyby dates and is able to add or delete deep-space maneuvers and powered flybys to find an optimal solution.

Trajectories to Asteroids

As seen in Fig. 2 and discussed earlier, 3:2 and 4:3 ΔV-EGAs have a performance advantage over the other types of ΔV-EGAs for final aphelion radii less than about 4.3 AU, which encompasses the orbits of most of the asteroids. For this reason we choose to apply these ΔV-EGAs to missions to asteroids. We select as our primary target the fourth largest asteroid, Hygiea (radius > 200 km). There are no larger asteroids beyond the orbit of Hygiea, which has a semimajor axis of 3.1 AU and an eccentricity of 0.12.

As a preliminary step to the ΔV-EGA trajectories, we investigate direct trajectories from Earth to Hygiea. Using STOUR, we searched for these direct trajectories over a 6-year span from 2000 through 2005. We then used MIDAS to minimize the launch energy. The trajectory with the minimum launch energy over the 6-year span has a launch date on Dec. 28, 2004, and a launch V_∞ of 6.36 km/s. The transfer orbit passes through aphelion (at 2.86 AU) and intercepts Hygiea near the ecliptic plane more than 6 months past its perihelion. The flight time of 1.4 years can be reduced to less than 1 year with a launch V_∞ of over 7 km/s.

We now add 3:2 ΔV-EGAs to the beginning of the direct trajectory to reduce the required launch energy and the total mission ΔV. The analytic predictions for 3:2 ΔV-EGAs, with final aphelion radii of 2.86 AU, and the MIDAS results are presented in Table 1. The

Table 1 Trajectories to Hygiea, 3:2 ΔV-EGA

	3:2(1)$^+$		3:2(1)$^-$		3:2(2)$^+$		3:2(2)$^-$	
	Analytic	MIDAS	Analytic	MIDAS	Analytic	MIDAS	Analytic	MIDAS
Launch V_∞, km/s	3.60	3.56	3.51	3.46	3.47	3.43	3.36	3.33
Aphelion ΔV, km/s	0.471	0.458	0.497	0.483	0.508	0.492	0.540	0.523
Days from aphelion	0	4.7	0	5.2	0	5.1	0	4.9
Earth to Earth time of flight, years	3.13	3.13	2.86	2.87	3.14	3.13	2.86	2.86

Table 2 Injected payload mass[11] to Hygiea

Launch vehicle	Direct trajectory, kg	3:2(2)$^-\Delta V$-EGA, kg	Cost (Ref. 12), \$M, RY 1999
Pegasus XL/Star 27	40	80	20[a]
Delta II 7925	550	1000	54
Atlas IIAS	1000	2200	105–145

[a]Excluding Star 27.

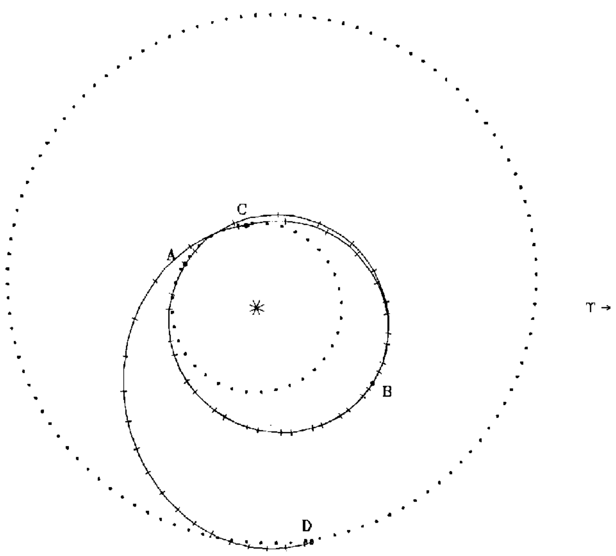

Fig. 7 Trajectory to Hygiea, 3:2(2)$^-$ ΔV-EGA. Event times: A, Launch, Feb. 17, 2002; B, ΔV, May 23, 2004; C, Earth gravity assist, Dec. 28, 2004; and D, Hygiea, June 8, 2006. 30-day ticks on s/c.

predictions from the analytic theory and the optimized results agree quite well. The final optimized trajectory in each case has only one maneuver, and this maneuver is within about 5 days of an aphelion passage.

The 3:2(2)$^-$ ΔV-EGA to Hygiea is shown in Fig. 7. We note that the return orbit of the − type ΔV-EGA with the maneuver on the last revolution does not pass through perihelion before the Earth encounter, and so the minimum radius from the sun of these orbits is not much less than 1 AU. (This is an important consideration from a spacecraft design point of view because trajectories with low perihelia require additional thermal protection for the spacecraft.) The portion of the trajectory from the Earth flyby to Hygiea (from C to D in Fig. 7) is nearly identical to the direct trajectory with minimum launch energy described earlier.

The 3:2(2)$^-$ ΔV-EGA in Fig. 7 allows us to reduce the launch V_∞ to 3.33 km/s from 6.36 km/s for a direct launch. To illustrate the advantage of this reduction we consider the payload performance of several launch vehicles. Injected payload mass capabilities[11] for the direct and ΔV-EGA trajectories are presented in Table 2. Thus, for example, a Delta II 7925 can launch a 550-kg spacecraft on the direct trajectory or a 1000-kg spacecraft on the ΔV-EGA trajectory. From another point of view, a 1000-kg spacecraft can be launched on the direct trajectory with an Atlas IIAS, costing \$105−\$145 million in 1999 real year (RY) dollars[12] or on the ΔV-EGA trajectory with a Delta II 7925, costing \$54 million, resulting in a savings of over \$50 million. The tradeoff is the additional mass required to execute a postlaunch (aphelion) ΔV of 0.523 km/s on the ΔV-EGA trajectory and an increase in flight time of 2.86 years. The total ΔV is reduced from 4.92 to 4.23 km/s. Although this may seem like a relatively small difference, we note that propellant mass increases exponentially with ΔV.

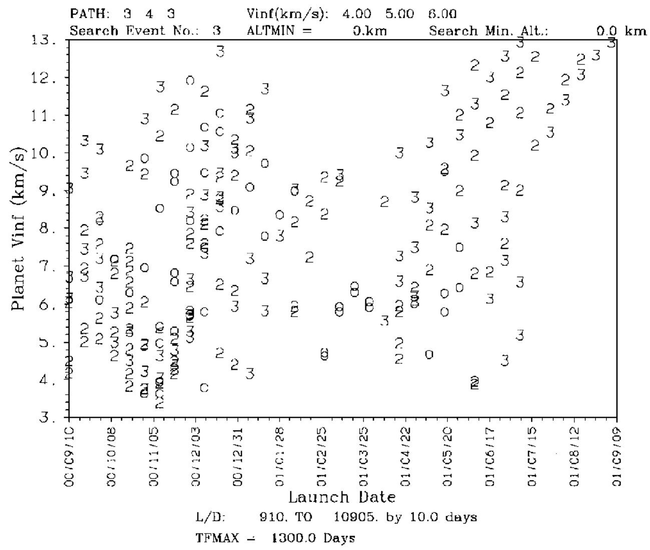

Fig. 8 V_∞ leveraging using Mars.

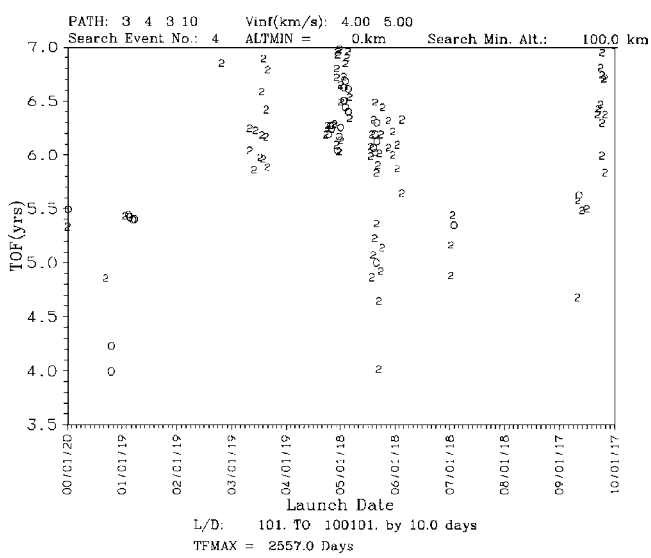

Fig. 9 Earth–Mars–Earth–Hygiea trajectories, 2000–2010.

Mars Gravity Assist

The aphelion of the nominal orbit of the 3:2 ΔV-EGA is 1.6 AU, slightly larger than the semimajor axis of Mars (1.52 AU). Therefore, a Mars flyby would occur near aphelion and could be used to offset or entirely replace the aphelion ΔV, thereby making these trajectories very efficient in terms of propellant usage. The effectiveness of this type of V_∞ leveraging is shown in Fig. 8. This figure gives the V_∞ at Earth return vs the launch date for trajectories that are launched from Earth, fly by Mars, and return to Earth. (An explanation of the nomenclature in Figs. 8 and 9 is presented in

Table 3 Legend for trajectory data vs launch date plots

PATH	Sequence of planets encountered. Numbers larger than 9 represent nonplanetary bodies. For example, PATH: 3 4 3 10 implies Earth–Mars–Earth–Asteroid (see Fig. 9).
Search event no.	Event in PATH sequence for which plot is made. For example, Search Event No.: 4 implies that the plot in Fig. 9 is for the asteroid arrival (for the path 3 4 3 10).
V_{inf}	Launch V_∞. The numbers in the plot (0, 2, 3, . . .; where 0 is used in lieu of 1 because it is more easily distinguished) represent these launch V_∞. For example, the numeral 3 on the plot in Fig. 8 represents a launch V_∞ of 6.00 km/s.
ALTMIN	Minimum flyby altitude allowed in the original file.
Search min. alt.	Minimum flyby altitude allowed for trajectories in the plot.
L/D	Range of launch dates where 910 represents Sept. 10, 2000, and 10905 represents Sept. 5, 2001 (Fig. 8). The launch date increment is also given, for example, by 10.0 days.
TFMAX	Maximum allowable time of flight.

Table 4 Trajectories to Mercury, 3:4 Δ V-EGA

	3:4(1)$^+$		3:4(1)$^-$		3:4(4)$^+$		3:4(4)$^-$	
	Analytic	MIDAS	Analytic	MIDAS	Analytic	MIDAS	Analytic	MIDAS
Launch V_∞, km/s	4.09	4.12	3.94	3.90	3.58	3.60	3.35	3.29
Perihelion ΔV, km/s	0.448	0.464	0.487	0.513	0.591	0.610	0.665	0.701
Days from perihelion	0	3.8	0	1.2	0	2.0	0	0.7
Earth to Earth time of flight, years	3.09	3.09	2.91	2.91	3.11	3.11	2.88	2.88

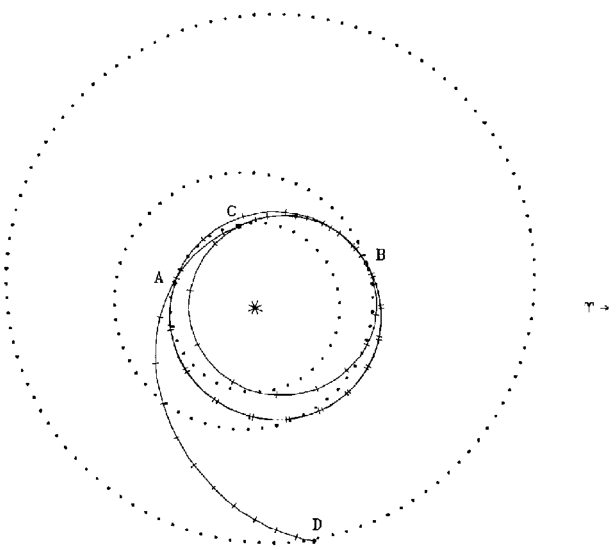

Fig. 10 Earth–Mars–Earth–Hygiea trajectory, 4:3(2)$^-$ Δ V-EGA analog. Event times: A, Launch, March 3, 2007; B, Mars gravity assist, July 3, 2009; C, Earth gravity assist, Jan. 2, 2011; and D, Hygiea, Jan. 2, 2012. 30-day ticks on s/c.

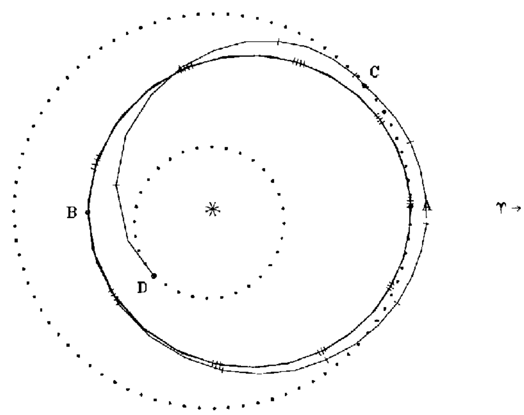

Fig. 11 Trajectory to Mercury, 3:4(4)$^+$ Δ V-EGA. Event times: A, Launch, Sept. 23, 1998; B, Δ V, April 20, 2001; C, Earth gravity assist, Nov. 1, 2001; and D, Mercury, Feb. 22, 2002. 30-day ticks on s/c.

Table 3.) In some cases, trajectories with a launch V_∞ of 4 km/s return to Earth with a V_∞ of 12 km/s, and those with a launch V_∞ of 5 or 6 km/s return with a V_∞ of around 13 km/s. This is significant leveraging without any deterministic postlaunch maneuvers. These results strongly suggest that a Mars gravity assist can be used in place of the aphelion ΔV to construct low ΔV trajectories to the asteroids.

With this in mind, we use STOUR to perform a broad search for such trajectories to Hygiea. Figure 9 presents trajectories that are launched from Earth, fly by Mars, return to Earth, and then proceed to Hygiea. Some of these trajectories are similar to a 3:2ΔV-EGA, whereas others resemble a 4:3ΔV-EGA. In each case the Mars flyby replaces the aphelion ΔV, and there are no deterministic postlaunch maneuvers. A trajectory analogous to a 4:3(2)$^-$ ΔV-EGA is shown in Fig. 10. It has a launch V_∞ of 2.95 km/s, flies by Mars on the second revolution, and reaches Hygiea in 4.8 years. A trajectory that resembles a 3:2(2)$^-$ ΔV-EGA is presented in Ref. 7. The launch V_∞ is 3.73 km/s with a flight time of 4.1 years.

The trajectory in Fig. 10 allows us to reduce the launch V_∞ from 6.36 km/s for the direct trajectory with minimum launch energy, discussed earlier, to 2.95 km/s without any deterministic postlaunch maneuver. The injected payload mass capabilities are even larger than that of the 3:2(2)$^-$ ΔV-EGA presented in Table 2. Inasmuch as no major postlaunch ΔV is required, nearly all of the additional injected payload mass capability can be used to enhance the scientific payload. The flight time in this case is 3.4 years longer than the direct trajectory.

Trajectories to Mercury

As shown in Fig. 2, each of the interior ΔV-EGAs that are plotted can potentially reach Mercury. We will compare the analytic results (which assume a circular Earth orbit) to trajectories obtained with MIDAS (which includes a more realistic Earth orbit). As before, the first step is to investigate direct trajectories. The trajectory with the minimum launch energy in 2001 has a launch date on Nov. 1, 2001, and a launch V_∞ of 6.39 km/s. It passes through perihelion (at 0.45 AU) just prior to reaching Mercury.

We now add 3:4ΔV-EGAs to the beginning of the direct trajectory to reduce the required launch energy and the total mission ΔV. The analytic predictions for 3:4 ΔV-EGAs, with final perihelion radii of 0.45 AU, and the MIDAS results are presented in Table 4. The predictions from the analytic theory and the optimized results agree quite well. Each optimized trajectory in Table 4 has only one maneuver, and the maneuver is less than 4 days from a perihelion passage.

The 3:4(4)$^+$$\Delta V$-EGA to Mercury is shown in Fig. 11. The portion of the trajectory from the Earth flyby to Mercury (from C to D in

Table 5 Trajectory to Mercury, 3:4 $(1, 4)^-$ ΔV-EGA

Launch V_∞, km/s	3.49
First orbit revolution	
Perihelion ΔV, km/s	0.184
Days from perihelion	2.0
Fourth orbit revolution	
Perihelion ΔV, km/s	0.452
Days from perihelion	1.9
Earth to Earth time of flight, years	2.89

Fig. 11) is nearly identical to the direct trajectory with minimum launch energy already described. The 3:4$(4)^-$ ΔV-EGA allows us to reduce the launch V_∞ to 3.29 km/s from 6.39 km/s for a direct launch. This reduction is very similar to that achieved in using a 3:2$(2)^-$ ΔV-EGA to reach Hygiea (already described) although the postlaunch ΔV is larger for the trajectory to Mercury.

The leveraging ΔV can be distributed among more than one perihelion, breaking a single maneuver into a series of smaller maneuvers. This process allows us to fine tune the allocation of total ΔV between launch ΔV and postlaunch ΔV within a particular $K{:}L^\pm$ type ΔV-EGA. As an example, in Table 5 we present the characteristics of a locally optimum 3:4^- ΔV-EGA to Mercury with leveraging maneuvers on the first and fourth spacecraft orbit revolutions. The values of launch V_∞, total leveraging ΔV, and flight time are between those of the 3:4$(1)^-$ and 3:4$(4)^-$ ΔV-EGAs in Table 4. Similarly, the leveraging ΔV can be distributed among more than one aphelion for multiple-revolution exterior ΔV-EGAs.

Conclusion

The generalization of ΔV-EGA trajectories provides new mission design concepts, which open the door to low-cost missions to the asteroids, Mercury, and other targets of interest. The V_∞ leveraging technique can reduce launch-energy requirements tremendously, at the cost of a deep-space maneuver and increased flight time. In some cases the deep-space maneuver can be eliminated altogether by replacing it with a gravity-assist maneuver, thus providing an even lower total ΔV. These concepts can potentially reduce launch costs by tens of millions of dollars or more.

Acknowledgments

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